

Regular Inverse Secure Domination in Graphs

R.Abinaya¹, P.Poongodi²

M.Phil Scholar, Dept. of Mathematics, SCAS, Dgl, Tamil Nadu, India¹

Asst. Professor, Dept. of Mathematics, SCAS, Dgl, Tamil Nadu, India²

ABSTRACT: In this paper, we have introduced regular inverse secure domination in graph and then discussed some standard theorems and examples of regular inverse secure domination in graph.

KEYWORDS: Domination, Secure domination, Inverse secure domination, Regular set domination, Regular set domination number.

I. INTRODUCTION

All graphs considered in this paper are finite, undirected with no loops or multi edge. Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962. That domination in graphs became an area of study of many researchers. One type of domination parameter is the secure domination in graphs. This was studied and introduced by E.J.Cockayne et al. The inverse domination in graph was first found in the paper of Kulli. In his study we introduce the new domination parameter, the regular inverse secure domination in graphs, and give some important results.

II. PRELIMINARIES

Definition 2.1: Let $G = (V, E)$ be a graph. A Set $S \subseteq V$ is called a *dominating set* of G if every vertex in $V \setminus S$ is adjacent to a vertex in S . A dominating set S is called a *minimal dominating set* if $S \setminus \{v\}$ is not a dominating set for all $v \in S$. The minimum cardinality of a minimal dominating set of G is called the *domination number* of G and is denoted by $\gamma(G)$.

Definition 2.2: A Subset S of V is called an *independent set* if no two vertices in S are adjacent. The minimum cardinality of a maximal independent set is called the *independent domination number* of G and is denoted by $i(G)$.

Definition 2.3: A dominating set S in G is called a *secure dominating set* in G if for every $u \in V(G) \setminus S$, there exists $v \in S \cap N_G(u)$ such that $(S \setminus \{v\}) \cup \{u\}$ is a dominating set. The minimum cardinality of a secure dominating set is called the *secure domination number* of G and is denoted by $\gamma_s(G)$.

Definition 2.4: Let D be a minimum dominating set in G . The dominating set $S \subseteq V(G) \setminus D$ is called an *inverse dominating set* with respect to D . The minimum cardinality of inverse dominating set is called an *inverse domination number* of G and is denoted by $\gamma^{-1}(G)$.

Definition 2.5: Let C be a minimum secure dominating set in G . The secure dominating set $S \subseteq V(G) \setminus C$ is called an *inverse secure dominating set* with respect to C . The minimum cardinality of inverse secure dominating set is called an *inverse secure domination number* of G and is denoted by $\gamma_s^{-1}(G)$.

Definition 2.6: A Set D of vertices in a graph $G = (V, E)$ is said to be a *regular dominating set* if for every set $I \subseteq V - D$ there exists a nonempty set $S \subseteq D$ such that $\langle I \cup S \rangle$ is regular and for $|I| = 1$, $\langle I \cup S \rangle$ is 1-regular. The minimum cardinality of a regular set domination number is called the *regular set domination number* of G and is denoted by $\gamma_r(G)$.

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Definition 2.7: Let S be a subset of $V(G)$ and $v \in S$. Then the **private neighborhood** of v with respect to S set $Pn[v, S] = \{w \in V(G) : N[w] \cap S = \{v\}\}$.

Definition 2.8: A dominating set D of a graph G is a **maximal dominating set** if $V - D$ is not a dominating set of G . Maximal domination number $\gamma_m(G)$ of G is the minimum cardinality of a maximal dominating set.

III. REGULAR INVERSE SECURE DOMINATION IN GRAPHS

Definition 3.1: An inverse secure dominating set D is said to be **regular inverse secure dominating set** if for every $I \subseteq V - D$ there exists a non empty set $S \subseteq D$ such that $\langle I \cup S \rangle$ is regular. The minimum cardinality of a regular inverse secure dominating set is called the **regular inverse secure domination number**.

Example 3.1:

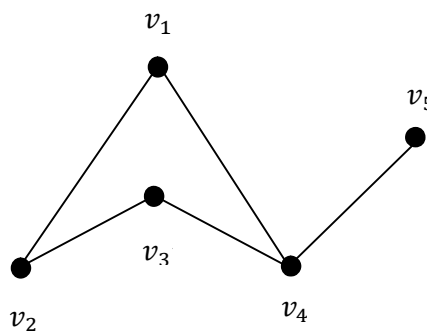


Fig. 1

$\{v_2, v_3, v_5\}$ is a regular inverse secure dominating set.

Theorem 3.1: [4] Let G be a graph with no isolated vertex. If $S \subseteq V(G)$ is a regular inverse secure dominating set then $V(G) \setminus S$ is also a regular inverse secure dominating set.

Proof: Let G be a graph with no isolated vertex and $S \subseteq V(G)$ is a regular inverse secure dominating set. To prove that $V(G) \setminus S$ is also a regular inverse secure dominating set. Let there exists a regular inverse secure dominating set in some graph G . Since the regular inverse secure dominating set of any graph G of order n cannot be $V(G)$. It follows that $\gamma_{rs}^{-1}(G) \neq n$ and hence $\gamma_{rs}^{-1}(G) < n$. Since $\gamma_{rs}^{-1}(G)$ does not always exists in a connected nontrivial graph G , we denote by G_s^{-1} be a family of all graphs with regular inverse secure dominating set. From this, we get $V(G) \setminus S$ is also a regular inverse secure dominating set.

Result 3.1: Let G be a connected graph of order $n \geq 4$. Then

- (i) $1 \leq \gamma_{rs}^{-1}(G) < n$
- (ii) $\gamma(G) \leq \gamma_{rs}^{-1}(G) \leq \gamma_s^{-1}(G)$

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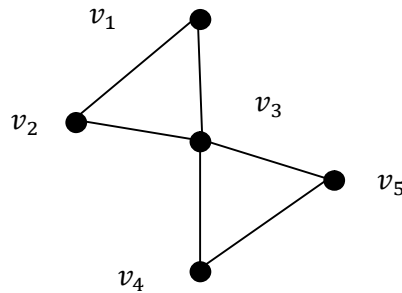


Fig. 2

- (i) Dominating Set = $\{v_3\}, \{v_2, v_4\}, \{v_1, v_3\}, \{v_2, v_3\}, \{v_5, v_3\}$

Minimum cardinality of the regular inverse dominating set is denoted by $\gamma_{rs}^{-1}(G)$

$$\gamma_{rs}^{-1}(G) = 2 \text{ and } n = 5$$

$$1 \leq 2 < 5$$

$$\text{Hence } 1 \leq \gamma_{rs}^{-1}(G) < n$$

- (ii) Minimum cardinality of the dominating set is called the domination number $\gamma(G)$

$$\gamma(G) = 1$$

Minimum cardinality of the regular inverse dominating set is denoted by $\gamma_{rs}^{-1}(G)$

$$\gamma_{rs}^{-1}(G) = 2$$

Minimum cardinality of the secure dominating set is denoted by $\gamma_s^{-1}(G)$

$$\gamma_{rs}^{-1}(G) = 2$$

$$1 \leq 2 \leq 2$$

$$\gamma(G) \leq \gamma_{rs}^{-1}(G) \leq \gamma_s^{-1}(G)$$

Theorem 3.2: [9] For any graph G , $\gamma_{rs}^{-1}(G) = 1$ if and only if $G = K_n$.

Proof: Let $\gamma_{rs}^{-1}(G) = 1$

To prove that, $G = K_n$. Let $D = \{v\}$ be a regular inverse secure dominating set of G . Suppose there exists two nonadjacent vertices $u, w \in V - D$. Then $\langle \{u, w, v\} \rangle$ is not regular, which is a contradiction. Hence every two vertices in $V - D$ are adjacent. Hence G is complete.

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Conversely,

Let G be a complete graph. To prove that, $\gamma_{rs}^{-1}(G) = 1$. We know that, "Let G be a connected non trivial graph of order n . Then $\gamma_{rs}^{-1}(G) = 1$ if and only if $G = K_n$." Every complete graph is a connected non trivial graph. Hence G is connected. Hence $\gamma_{rs}^{-1}(G) = 1$

Theorem 3.3: [9] Let G be a connected non complete graph of order $n \geq 4$, if $\gamma_{rs}^{-1}(G) = 2$ then $\gamma_{rs}(G) = 2$.

Proof: Let G be a connected non complete graph of order $n \geq 4$ and $\gamma_{rs}^{-1}(G) = 2$. To prove that, $\gamma_{rs}(G) = 2$. Suppose that $\gamma_{rs}^{-1}(G) = 2$. "Let G be a connected nontrivial graph of order $n \geq 4$. Then $\gamma_{rs}(G) \leq \gamma_{rs}^{-1}(G)$ ". Since $\gamma_{rs}(G) \leq \gamma_{rs}^{-1}(G) = 2$. It follows that, $\gamma_{rs}(G) = 1$ or $\gamma_{rs}(G) = 2$. If $\gamma_{rs}(G) = 1$ then $G = K_n$ Which is a contradiction. Hence $\gamma_{rs}(G) = 2$

Theorem 3.4: For any graph G , $\gamma_{rs}^{-1}(G) \leq n - \Delta(G)$

Proof: Let u be a vertex of maximum degree Δ . Let $D = V - N(u)$. Then $|D| = n - \Delta(G)$ and $v \in D$. Consider a maximal regular inverse secure dominating set S in $\langle D \rangle$ containing v . Then S is the regular inverse secure dominating set of $\langle D \rangle$. Since v dominates $N(v)$, S dominate $N(v)$. S is the regular inverse secure dominating set of G .

$$|S| \leq |D| = n - \Delta(G), \quad \gamma_{rs}^{-1}(G) \leq n - \Delta(G)$$

Corollary 3.4: For any graph G $\gamma_{rs}^{-1}(G) \leq n - k(G)$

Proof: $\gamma_{rs}^{-1}(G) \leq n - \Delta(G)$, $\gamma_{rs}^{-1}(G) \leq n - \delta(G)$

$$\text{Since } \delta(G) \leq \Delta(G)$$

$$\gamma_{rs}^{-1}(G) \leq n - k(G), [k(G) \leq \delta(G)].$$

$$\gamma_{rs}^{-1}(G) \leq n - k(G).$$

Theorem 3.5: [11] Let G be a regular graph with $diam(G) \leq 3$ then any regular inverse secure dominating set D of G is a set dominating set of G , where $diam(G)$ is the diameter of G .

Proof: Let G be a graph, with $diam(G) \leq 3$. To prove that, Any regular inverse secure dominating set D of G is a dominating set. Clearly, G is connected. Suppose there exists two non adjacent vertices $u, v \in V - D$ such that every $u - v$ path in G contains a vertex of $V - D$. Then there exists a vertex $w \in V - D$ adjacent to u or v , say u . Thus for the set $I = V - D - \{v\}$ there exists no nonempty set $S \subseteq D$ such that $\langle I \cup S \rangle$ is regular, a contradiction. Hence there exists a nonempty set $S \subseteq D$ such that $\langle \{u, v\} \cup S \rangle$ is connected. This implies that D is a set dominating set.

Theorem 3.6: [11] If u and v are two vertices adjacent to same cut vertex, then there is a γ_{rs}^{-1} set of G containing v .

Proof: Let D be a γ_{rs}^{-1} set of G . Suppose $u, v \in V - D$. Then there exists a vertex $w \in D$ not adjacent to any vertex of $V - D$. This implies that $D - \{w\} \cup \{v\}$ is a γ_{rs}^{-1} set of G .

IV. CONCLUSION

In this paper we have introduced the concept of regular inverse secure domination in graphs by using the concept of secure domination in graphs, inverse secure domination in graphs and regular domination in graphs.

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